Closing Tues: 5.1/5.2, 5.3

Closing Thur: Optional Ch. 2 Review

Exam 2 is Thursday, Feb. 27th

5.3: More About Exponentials and Logs

5.1/5.2 / PROBLEMS 1-5

PLUG IN NUMBERS!

PROBLEM 2

$$S = 1000\, e^{0.05n}$$

- PLUG IN $n = 0 \Rightarrow S = 1000$
- PLUG IN $n = 20 \Rightarrow S \simeq 1000\, e^{0.05(20)} \simeq 2718.28$

PICK THE GRAPH THAT GOES THRU $(0, 1000)$ AND $(20, 2718.28)$

PROBLEM 4 $f(x) = \ln(x)$

FIND $f(e^{6x}) = \ln(e^{6x}) = \boxed{6x}$

VERY QUICK! 4/5 ARE SEEING IF YOU UNDERSTAND INVERSES!

Entry Task: Do you know the basic algebra solving toolbox? Show me, solve these...

Equation	Inverse
$x + 3 = 14$	$x = 14 - 3 = 11$
$y - 5 = 22$	$y = 22 + 5 = 27$
$3t = 16$	$t = \frac{16}{3} = 5.\overline{3}$
$\frac{m}{0.2} = 100$	$m = 0.2 \cdot 100 = 20$
$x^2 = 7$ EVEN ROOT	$x = \pm\sqrt{7} \simeq \pm 2.64575$
$\sqrt{y} = 3$	$y = 3^2 = 9$
$t^5 = 20$ ODD ROOT	$t = \sqrt[5]{20} = 20^{(1/5)} \simeq 1.82056$
$\sqrt[5]{w} = 3$	$w = 3^5 = 243$
$e^x = 10$	$x = \ln(10) \simeq 2.302585$
$\ln(y) = 3$	$y = e^3 \simeq 20.0855$
$5^t = 60$	$t = \log_5(60) \simeq 2.543959\ldots$

(NOT IN YOUR CALCULATOR)

Another perspective: Many students think of inverses as "canceling".

$f(f^{-1}(x)) = x$

Examples:

1. $x - 5 = 22 \rightarrow \underbrace{x - 5 + 5} = 22 + 5$

$x = 27$

2. $3t = 16 \rightarrow \frac{3t}{3} = \frac{16}{3}$

$t = 5.\overline{3}$

3. $t^5 = 20 \rightarrow (t^5)^{\frac{1}{5}} = (20)^{\frac{1}{5}}$

$t \approx 1.82056\ldots$

4. $e^x = 10 \rightarrow \ln(e^x) = \ln(10)$

$x = \ln(10)$

5. $\ln(y) = 3 \rightarrow e^{\ln(y)} = e^3$

$y = e^3$

Solving is using inverses in correct order!

Directly from Homework:

5.1/5.2: Problem 8

Solve $\frac{1}{6}e^{-4t} = 0.9$

$\times 6$

$e^{-4t} = 5.4$

$\ln(\ )$

$-4t = \ln(5.4)$

$\div -4$

$t = \frac{\ln(5.4)}{-4}$

$t \approx \frac{1.6863989535\ldots}{-4}$

$t \approx -0.42159974\ldots$

CHECK!

ANOTHER WAY TO THINK ABOUT IT

$e^{-4t} = 5.4$

$\ln(e^{-4t}) = \ln(5.4)$

$\frac{-4t}{-4} = \frac{\ln(5.4)}{-4}$

$t = \frac{\ln(5.4)}{-4}$

SAME!

5.1/5.2: Problem 10

CHECK

Solve $8e^{2t-5} = 24$

$\div 8$

$e^{(2t-5)} = 3$

$\ln(\ )$

$\ln\left(e^{(2t-5)}\right) = \ln(3)$

$2t-5 \approx 1.098612288668$

$+5$

$2t \approx 6.098612288668$

$\div 2$

$t \approx 3.049306144...$

SAME

EXACT ANSWER: $t = \dfrac{\ln(3)+5}{2}$

5.3: Problem 4

Solve $77 = 100 - 100e^{-0.08x}$

-100

$-23 = -100e^{-0.08x}$

$\div(-100)$

$0.23 = e^{-0.08x}$

$\ln(0.23) = \ln\left(e^{-0.08x}\right) = -0.08x$

$\dfrac{\ln(0.23)}{-0.08} = x$

$x \approx 18.3709496...$

5.3: Problem 5

Solve $65 = \dfrac{75}{1+5e^{-0.3x}}$

"CLEAR DENOMINATOR!" MULTIPLY BOTH SIDES BY $(1+5e^{-0.3x})$

$\Rightarrow 65(1+5e^{-0.3x}) = 75$

$\div 65$

$1+5e^{-0.3x} = \frac{75}{65} \approx 1.153846154...$

-1

$5e^{-0.3x} \approx 0.153846154...$

$\div 5$

$e^{-0.3x} \approx 0.030769231...$

$\ln(\ )$

$-0.3x = \ln(0.030769231...)$

$-0.3x \approx -3.481240089...$

$\div -0.3$

$x \approx 11.6041336...$

Basic Logarithm Facts (SECTION 5.3)

Rule
$1 = e^0$ so $\ln(1) = 0$ $e = e^1$ so $\ln(e) = 1$
$\ln(ab) = \ln(a) + \ln(b)$
$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$
$\boldsymbol{\ln(a^b) = b\ln(a)}$
$\ln(e^x) = x$ $e^{\ln(y)} = y$

ASIDE PROOF OF THIS

$a = e^{\ln(a)}$, so $a^b = (e^{\ln(a)})^b = e^{b\ln(a)}$

THUS,

$\ln(a^b) = \ln(e^{b\ln(a)}) = b\ln(a)$

5.3 Problem 1:

Solve $6^{5x} = 56562$

$\log_6$ NOT IN CALCULATOR SO DO THIS ↘ $\ln(\ )$

$\ln(6^{5x}) = \ln(56562)$ (RULE)

$5x\ln(6) = \ln(56562)$

$\Rightarrow 5 \cdot x \cdot 1.791759\ldots \approx 10.94309266\ldots$

$x \approx \frac{10.94309266\ldots}{5 \cdot 1.791759\ldots} \approx \boxed{1.22149126\ldots}$ ÷5 AND ÷1.791759.

CHECK ✓

EXACT ANSWER (SAME) $x = \frac{\ln(56562)}{(5\cdot\ln(6))}$

5.3: Problem 2

Solve for x: $\frac{0.23P}{P} = \frac{P(2)^{-x}}{P}$ ÷P

$0.23 = (2)^{-x}$ ↘ $\ln(\ )$

$\ln(0.23) = \ln(2^{-x})$ (RULE!)

$\ln(0.23) = -x\ln(2)$

$\boxed{x = \frac{\ln(0.23)}{-\ln(2)} \approx 2.120294\ldots}$

CHECK ✓

The Ch. 5 homework just gives practice with plugging in and solving. Here are the "hard" problems:

5.3: Problem 7: Given $P = Ae^{-0.05t}$

(a) A = 75000, P = 40000, what is t?

(b) P = 60000, t = 10, what is A?

(a) $40000 = 75000e^{-0.05t}$

SOLVE! JUST LIKE WE HAVE DONE TODAY

(b) $60000 = A\underbrace{e^{-0.05(10)}}$

$60000 = A \cdot (0.6065306597\ldots)$

SOLVE, THIS IS EASIER, JUST DIVIDE!

$A = \frac{60000}{0.6065306597} \approx 9892.3276\ldots$

5.3: Problem 8: Given $y = P_0e^{ht}$

Given in 1998 (t=0), y = 100,000:

so $100000 = P_0e^{h(0)}$

Given in 2008 (t=10), y = 110,365:

so $110365 = P_0e^{h(10)}$

Asked: in 2023 (t=25), what is y?

$100000 = P_0\underbrace{e^0}_{1} = P_0$

So $\boxed{P_0 = 100000}$ ← USE THIS FOR THE REST OF THE PROBLEM

$110365 = 100000\,e^{10h}$ ÷ 100000

$1.10365 = e^{10h}$ ln( )

$\ln(1.10365) = 10h$

$\boxed{h = \frac{\ln(1.10365)}{10} \approx 0.0098622868\ldots}$ ← USE THIS FOR THE REST OF PROBLEM

So $y = 100000\,e^{0.0098622868t}$

PLUG IN $t = 25$!

$y = 100000\,e^{0.0098622868(25)}$

≈ 127961.2

$\approx$ 127961 PEOPLE

5.3: Problem 10: Given $p = 100e^{-\frac{q}{2}}$

(a) If q = 4, what is p?

(b) If p = 2.01, what is q?

(a) COMPUTE

$$p = 100e^{-\frac{4}{2}} \approx 13.53353\ldots$$

$$\approx \$13.53$$

NEAREST CENT

(b) SOLVE $2.01 = 100e^{-q/2}$

JUST LIKE WHAT WE LEARNED TODAY!

5.3: Problem 11:

Given $p = 500e^{-0.07x}$=price.

What is revenue when x = 100?

$$\text{PRICE} = 500e^{-0.07\cdot 100} \approx 0.45594098\ldots$$

$$\text{QUANTITY} = x = 100$$

$$\text{TR}(x) = (\text{PRICE})(\text{QUANTITY})$$

$$= 500e^{-0.07x} \cdot x$$

$$\text{TR}(100) = \underbrace{500e^{-0.07\cdot 100}}_{0.45594098} \cdot 100$$

$$\approx 45.594098$$

$$\approx \$45.59$$